Liquid drop model

Liquid drop model was proposed by Neils Bohr and F. Kalcar in the year 1937. They observed that there exists many similarities between the drop of a liquid and a nucleus. The nucleus is supposed to be spherical in shape in the stable state with radius $R_0 = r_0 A^{1/3}$ just as a liquid drop is spherical due to symmetrical surface tension forces. Each molecule of the liquid associates with those molecules touching it, but not with those molecules separated widely from it. This shows that the binding forces are of short range. Similarly, the binding forces of the nucleons in the nucleus are also of short range. Molecules of the liquid at the surface are less tightly bound because they are not surrounded. A surface effect reduces the binding energy and is a negative effect for the nucleus. There is also an electrostatic force of repulsion between the various protons in the nucleus and it is a negative effect. Therefore, the total binding energy is due to three effects. Hence the individual nucleons must be able to move about within nucleus much as does molecule of a liquid drop. Therefore they proposed ~~that~~ a model of nucleus like a small liquid drop. Such a model is thus known as liquid drop model.

The density of the nucleus is independent of the type of the nucleus and its value is $2.3 \times 10^{17}\ kg/m^3$.

The following analogies hold between a small drop of liquid and the nucleus:—

(i) The liquid drop is spherical due to surface tension. Nucleus is also assumed to be spherical in shape.

(ii) The density of the nucleus is independent of its volume and the density of the liquid is also independent of the volume.

(iii) The molecules of the liquid drop interact over short-ranges. Similarly, nucleon in the nucleus also interact only with their immediate neighbours.

(iv) The molecule of the drop moves over a small distance in the drop. Similarly, the nucleon also moves over a small distance in the nucleus.

(V) The molecule of the drop leaves the drop during evaporation or when its temperature is raised and it gains energy in this process. Similarly, when a nucleon in the nucleus gains energy, it can leave the nucleus. In this process, energy is given to the nucleus by bombarding particle and a nucleon is ejected out of the nucleus.

(vi) When a drop of water is allowed to oscillate, it breaks up into two smaller drop of equal size. The process of nuclear fission is similar and the nucleus breaks up into two smaller nuclei.

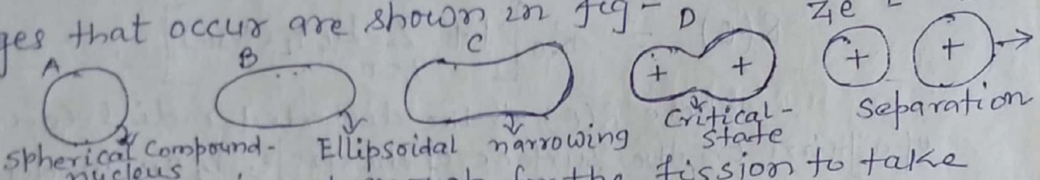
(vii) The force of surface tension acts on the surface of the liquid drop similarly, there is a potential barrier at the surface of the nucleus.

(viii) The condensation of drops bears semblance with the formation of compound nucleus.

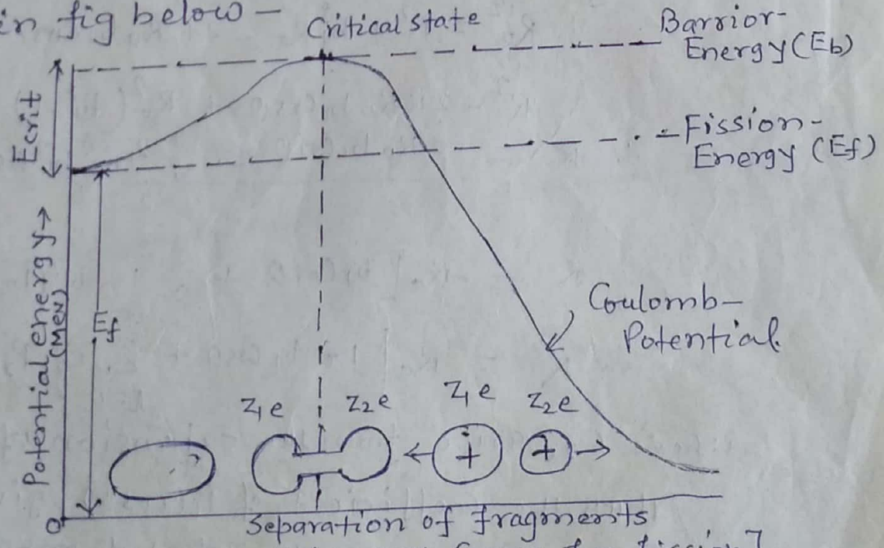
The process of nuclear fission has been successfully explained on the basis of the liquid drop model and it gives a collective picture of the nucleus and not of the inside of the nucleus.

Liquid drop theory of Nuclear fission
or
Bohr-Wheeler theory of Nuclear fission

N. Bohr and J.A Wheeler put forward the theory of nuclear fission based on the liquid drop model of the nucleus in 1939. They assumed that the nucleus to be drop of an incompressible, electrically charged nuclear fluid which when not under the action of any external force i.e in equilibrium adopts a spherical shape. Then they pictured the fission process as a result of the vibrations induced in this liquid drop by the incident particle, thus deforming its spherical shape and ultimately leading to its break up. The nuclear forces are short range, charge-independent nucleon-nucleon forces. The forces present within a nucleus are (i) nuclear forces (ii) the Coulomb repulsive forces between the protons. Nuclear forces which bind the nucleons together are compared with the surface tension forces of the liquid which also hold the molecules of the drop bound together. The nuclear drop may be made to disrupt by causing it to vibrate with large enough amplitude. The energy required to initiate these vibrations is supplied by the absorbed-neutron. The introduction of this excitation energy into the nuclear system profoundly disturbs the equilibrium shape of nucleus. The spherical shape changes to ellipsoidal. If the vibrations are of sufficient amplitude the ellipsoidal narrows to a 'dumb-bell' and finally breaks into two intermediate nuclei. The different stages that occur are shown in fig –



In case, energy is not enough for the fission to take place, the compound nucleus which is formed as a result of absorption of the incident neutron, returns back to its spherical shape by emitting either the neutron or γ-rays. There is a critical energy called <u>fission ~~three~~-threshold energy</u> or the <u>activation energy</u> which must be reached for complete separation. The critical energy is shown in fig below –



[Potential energy Curve for fission]

For the mathematical analysis, considering a nucleus to be initially of spherical shape, which is made up of incompressible nuclear fluid has a constant volume. On account of this incompressibility the vibrations set up in the nucleus due to the absorption of a neutron and can not penetrate any deep within the drop but shall remain confined to the surface of the drop only. Of course the deformation produced in the nucleus may be of any type but we assume for the sake of mathematical splicity that in the lowest excitations, the drop will still retain axial symmetry. Let us choose the axis of symmetry as the polar axis of the spherical co-ordinates.

On account of the axial symmetry, the deformation may be expressed as —

$$R(\theta) = R_0\left[1+\sum_{l=0}^{\infty} b_l P_l(\cos\theta)\right]; \quad \text{—— (1)}$$

where $R(\theta)$ is the distorted radial co-ordinate of a point on the surface of the drop, b_l are the deformation parameters and $P_l(\cos\theta)$ are the Legendre Polynomials.

This formula can give both a deformation of the sphere as well as a translation of the sphere as a whole. But here the translation of the sphere is not taken into consideration. To judge the correctness of the above statement, considering the sphere to be moved undeformed through a distance $b_1 R_0$ along the axial line ($\theta = 0$), then we have

$$R_0^2 = R^2 + b_1^2 R_0^2 - 2RR_0 b_1 \cos\theta$$

$$R^2 - 2RR_0 b_1 \cos\theta + R_0^2(b_1^2 - 1) = 0$$

$$\therefore R = \frac{2R_0 b_1 \cos\theta \pm \sqrt{4R_0^2 b_1^2 \cos^2\theta - 4R_0^2(b_1^2-1)}}{2}$$

$$R = R_0\left[b_1\cos\theta \pm \sqrt{1 - b_1^2(1-\cos^2\theta)}\right]$$

$$R = R_0\left[1 + b_1\cos\theta + \sum_{l=0}^{\infty} C_{2l} P_{2l}\cos\theta\right]; \quad \text{—— (2)}$$

where C_{2l} arise from the expansion of the square root.

Thus the coefficient of $P_1(\cos\theta)$ gives the distance through which the centre of drop moves and for a pure deformation without any translation $b_1 = 0$. Moreover b_0 is to be chosen in such a manner that the nuclear volume remains constant, we choose $b_0 = 0$.

Equation (1) becomes —

$$R(\theta) = R_0\left[1 + b_2 P_2(\cos\theta) + b_3 P_3(\cos\theta) + \cdots\right]; \quad \text{—— (3)}$$

Here the surface energy of a spherical drop is defined as

$$E_{S_0} = ST = 4\pi R_0^2 T \; ; \text{———} (4)$$

Where S is the surface area of the drop and T is the surface tension.

The surface energy of a deformed drop is given by

$$E_S = T\int dS \; ; \text{———} (5)$$

According to Bohr and Wheeler, the surface energy of a deformed nuclear drop in terms of the deformed parameters b_l may be written as

$$E_S = 4\pi R_0^2 T\left(1 + \frac{2}{5}b_2^2 + \frac{5}{7}b_3^2 + \cdots\cdots\right) \text{———} (6)$$

The Coulomb energy for an undeformed drop is given by

$$E_{C_0} = \frac{3}{5}\frac{(Ze)^2}{4\pi\epsilon_0 R_0} \; ; \text{———} (7)$$

In case of a deformed drop, the Coulomb energy in terms of the deformation parameters b_l is given by

$$E_C = \frac{3}{5}\cdot\frac{(Ze)^2}{4\pi\epsilon_0 R_0}\left[1 - \frac{1}{5}b_2^2 - \frac{10}{49}b_3^2 - \cdots\cdots\right] \; ; \text{———} (8)$$

Therefore, the total deformation energy E_T is obtained by adding (6) and (8)

$$E_T = E_S + E_C = 4\pi R_0^2 T\left(1 + \frac{2}{5}b_2^2 + \frac{5}{7}b_3^2 + \cdots\right) + \frac{3}{5}\frac{(Ze)^2}{4\pi\epsilon_0 R_0}\left[1 - \frac{1}{5}b_2^2 - \frac{10}{40}b_3^2 - \cdots\right] \; ; \text{———} (9)$$

The total energy in the undeformed state is equal to

$$E_{S_0} + E_{C_0}$$

Hence energy change due to deformation of the drop is given by

$$\Delta E = (E_S + E_C) - (E_{S_0} + E_{C_0}) \; ; \text{———} (10)$$

If we consider lowest excitations only i.e $b_2 \neq 0$ and $b_l = 0$ for $l > 2$, then the energy change due to the deformation of the drop is given by

$$\Delta E = (E_S + E_C) - (E_{S_0} + E_{C_0})$$

$$= E_{S_0}\left(1 + \frac{2}{5}b_2^2\right) + E_{C_0}\left(1 - \frac{1}{5}b_2^2\right) - (E_{S_0} + E_{C_0})$$

$$= \frac{2}{5}E_{S_0}b_2^2 - \frac{1}{5}E_{C_0}b_2^2$$

$$\Delta E = \frac{1}{5}b_2^2\left[2E_{S_0} - E_{C_0}\right] \; ; \text{———} (11)$$

This shows that the surface energy appears with a positive sign and Coulomb energy with negative sign.

Therefore,

(i) If $2E_{S_0} > E_{C_0}$, ΔE is +ve and since surface energy prevents disruption while the Coulomb energy promotes it. The nucleus will be stable against spontaneous decay.

(ii) If $2E_{S_0} < E_{C_0}$, ΔE becomes −ve and the nucleus will be unstable against spontaneous decay.

(iii) If $2E_{S_0} = E_{C_0}$, we have the critical case. In this case

$$2 \cdot 4\pi R_0^2 T = \frac{1}{4\pi\epsilon_0} \cdot \frac{3}{5} \cdot \frac{(Ze)^2}{R_0}$$

or $$8\pi r_0^2 A^{2/3} T = \frac{1}{4\pi\epsilon_0} \cdot \frac{3}{5} \frac{(Ze)^2 A^{-\frac{1}{3}}}{r_0} \quad \text{Since } R_0 = r_0 A^{\frac{1}{3}}$$

or $$Z^2/A = 4\pi\epsilon_0 \cdot \frac{40\pi r_0^2 T}{3e^2} \doteqdot 50 \quad ; \text{———} (12)$$

which defines a critical value of Z^2/A and accordingly we write $[Z^2/A]_{crit}$.

Fissionable Parameter(χ) can be defined as —

$$\chi = \frac{[Z^2/A]}{[Z^2/A]_{crit}} \quad ; \text{———} (13)$$

If $\chi > 1$, the nucleus is unstable against spontaneous fission. Critical energy (E_f) for fission is defined as the energy necessary to deform a drop when it is about to split into two equal drops, then may be written as —

$$E_f = 4\pi r_0 T A^{2/3} f(\chi) = E_{S_0} A^{2/3} f(\chi) \quad ; \text{———} (14)$$